MEASUES OF DISPERSION

Measures of dispersion-Range, Mean deviation, standard deviation-their relative measures(coefficients)

Time series- components- measuring trend by the method of moving average and method of least squares

Time series analysis

- It is a method in which the data relating to different time period is analysed to take business decisions.
- Trend: it is the direction of change
- Types: seasonal- changes according to seasons (demand for ice cream increases during summer and decreases during monsoon)
- Cyclical: changes according to the phases of the business cycle(boom, recession, depression and recovery)
- Secular: changes once in a very long period (plague and Now Corona)
- Irregular: there is no regular duration (unexpected, uncertain)

Measures of trend

1. Graphical method
2. Moving average method method of leastsquare

Graphical method

- The given data is plotted on the graph sheet along the OX and the OY Axis
- Time is taken on the OX axis and the quantity is taken on the OY axis.
- The corresponding points are marked and joined with straight line.
- This given the actual line.
- When the actual line is extended for the future time period, we get the trend line.
- The trend line is joined with dotted line to the OY axis to measure the value.

Moving average method

1. semi-average method
2. Even period moving average
3. Odd period moving average

Semi average method

- The given problem ids divided into two equal parts
- If the number is odd, leave the middle year.
- Average is calculated for both the parts.
- The given figures are plotted on the graph and joined with straight line and marked as actual line.
- The two averages are platted on the graph and joined with dotted lines and marked as trend line. This line can be extended.

Example

1.

Year	Sales	total	average
2006	41	129	49
2007	48		
2008	40		
2009	60	159	59
2010	40		
2011	59		

Example 2.

Year	Sales	total	average
2006	40		
2007	48		
2008	38	126	42
2009	60		
2010	50		
2011	56		
2012	50	156	52

Moving average method

- Odd period moving average: eg- 3 yearly, 5 yearly, 7 yearly etc
- The year for moving depends on the length of the data.

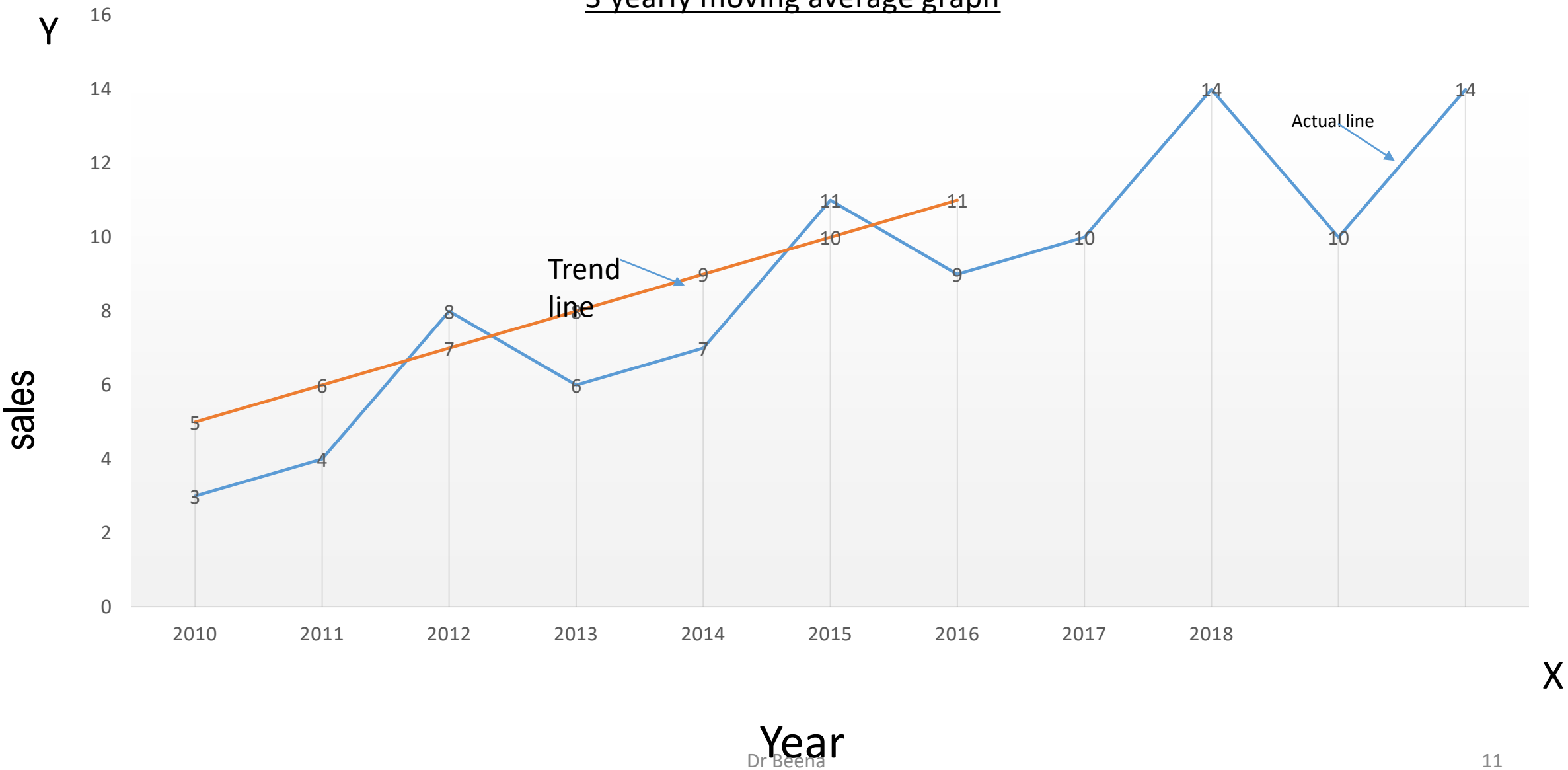
Example : 3 yearly moving average

1. Calculate 3 yearly moving average from the following:

year	sales
2010	3
2011	4
2012	8
2013	6
2014	7
2015	11
2016	9
2017	10
2018	14

year	sales	Three yearly total	Three yearly average
2010	3		
2011	4	15	5
2012	8	18	6
2013	6	21	7
2014	7	24	8
2015	11	27	9
2016	9	30	10
2017	10	33	11
2018	14		

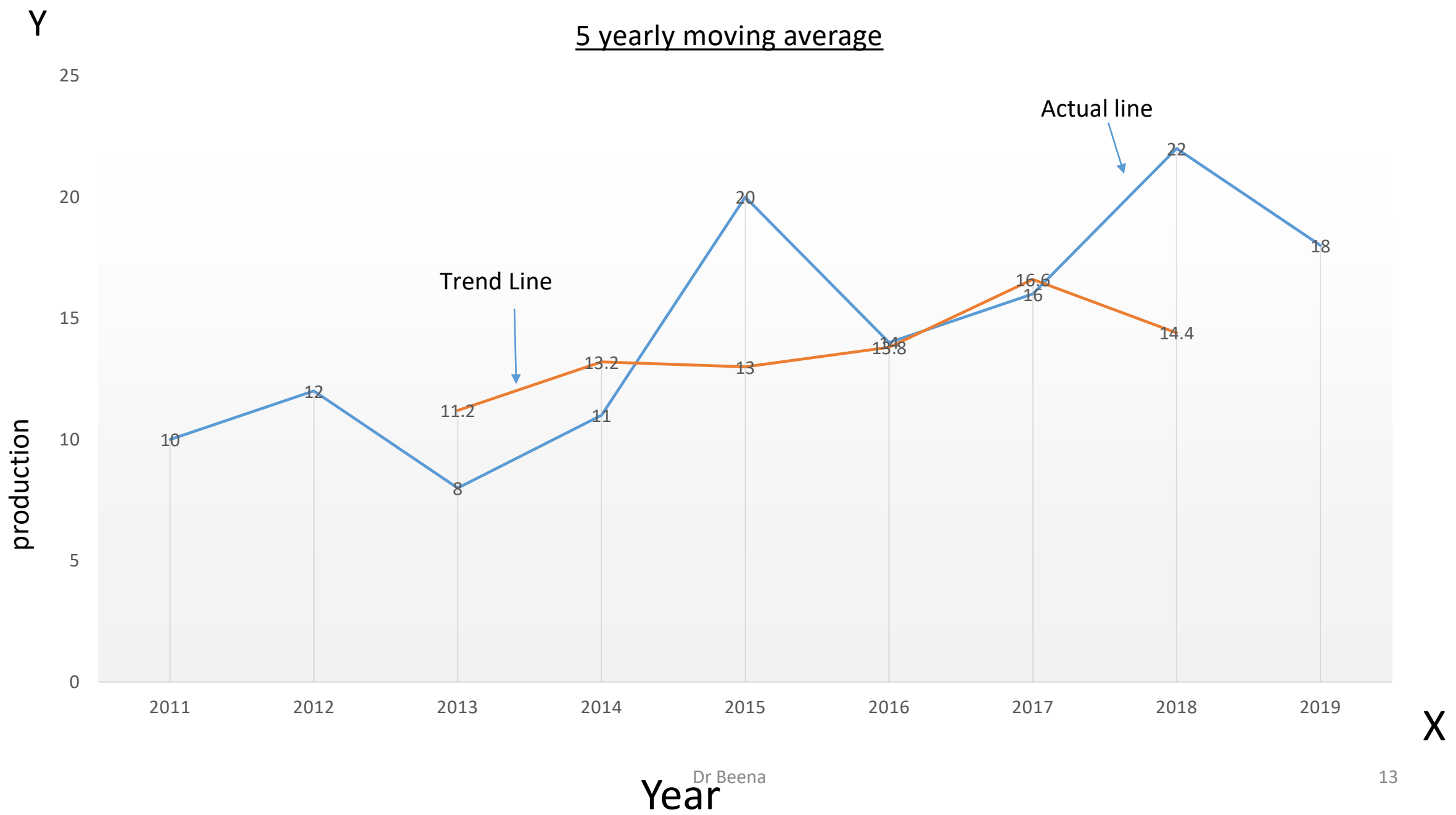
3 yearly moving average graph



Example 2.- 5 yearly moving

year	production
2011	10
2012	12
2013	08
2014	11
2015	20
2016	14
2017	16
2018	22
2019	18

year	production	5 yearly total	5 yearly average
2011	10		
2012	12		
2013	08	56	11.2
2014	11	66	13.2
2015	20	65	13
2016	14	69	13.8
2017	16	83	16.6
2018	22	72	14.4
2019	18		


Y
5 yearly moving average
25
20
15
10
5
0
Actual line
Trend Line
22
20
18
16.6
16
14.4
14
13.8
13.2
13
12
11.2
11
10
8
production
2011
2012
2013
2014
2015
2016
2017
2018
2019
X
Year

Even period moving average

- 2 yearly, 4 yearly, 6 yearly etc
- Cantering: it is a process of taking a two yearly average of the averages to get the centre year for plotting the averages.

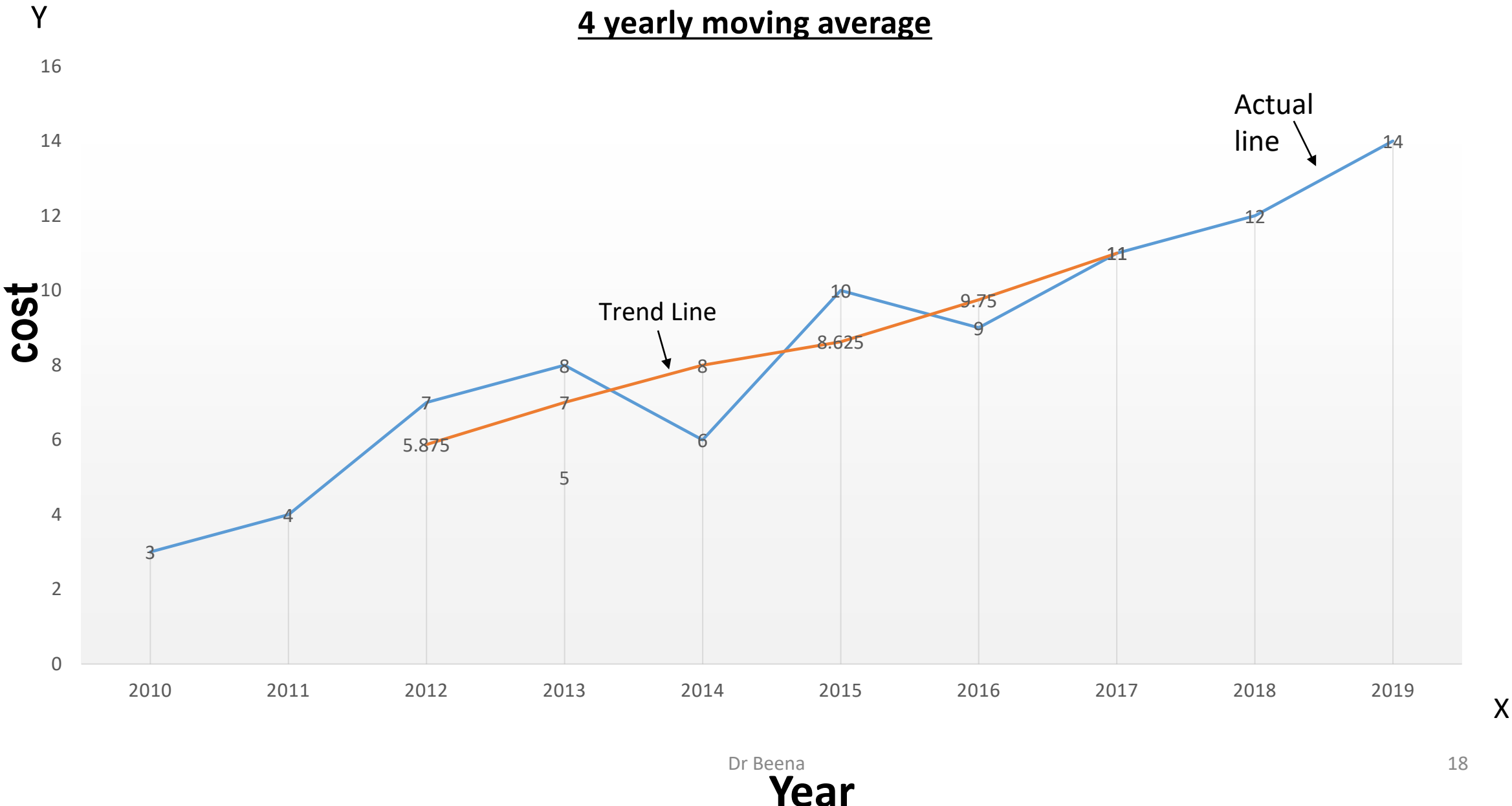
year	cost
2006	12
7	10
8	08
9	13
10	06
11	14
12	09
13	10
14	11
15	07

Example:

1. Calculate a four yearly average for the following:

year	Cost
2010	3
2011	4
2012	7
2013	8
2014	6
2015	10
2016	9
2017	11
2018	12
2019	14

year	Cost	4 yearly total	4 yearly average	2 yearly total of averages	Cantered average
2010	3	-	-	-	-
2011	4	22	5.5	-	-
2012	7	25	6.25	11.75 /2	5.875
2013	8	31	7.75	14 /2	7
2014	6	33	8.25	16 /2	8
2015	10	36	9	17.25 /2	8.625
2016	9	42	10.5	19.5 /2	9.75
2017	11	46	11.5	22 /2	11
2018	12	-	-	-	-
2019	14	-	-	-	-


4 yearly moving average
Y
cost
16
14
12
10
8
6
4
2
0
Actual line
Trend Line
3
4
7
8
6
10
9
11
12
14
5.875
7
5
8
8.625
9.75
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
X
Year

Method of least square

- It is a method where Y_o, the trend value for different years are calculated by using the formula,

$$Y_o=a+bX$$

The values of ‘a’ and ‘b’ are constant and can be derived from the following equations

$\sum Y= Na+b\sum X$ and

$\sum XY=a\sum X+b\sum X^2$

Steps:

1. Take the second column of the problem as Y
2. Get the X column for all the given years (Year – Mid year)
3. Multiply the values of X with the values of Y and obtain $\sum XY$
4. Get the total of X column- $\sum X$
5. Square X values and get the total - $\sum X^2$
6. Apply the equation and solve them.

Example

1. Find trend values for the following by the method of least square.

year	Y
2012	40
2013	52
2014	60
2015	78
2016	35
2017	50
2018	63
2019	90

year	Y	X (year-mid year)	XY	X^2	Y_o
2012	40	-3.5	-140	12.25	446.675
2013	52	-2.5	-130	6.25	48.625
2014	60	-1.5	-90	2.25	52.575
2015	78	-0.5	-39	0.25	56.525
2016	35	0.5	17.5	0.25	60.475
2017	50	1.5	75	2.25	64.425
2018	63	2.5	157.5	6.25	68.375
2019	90	3.5	315	12.25	72.325
TOTAL	468	0	166	42	

Working Note

- Mid year is taken in between 2015 and 2016 =2015.5
- $Y_o=a+bX$

$\sum Y= Na+b\sum X$

$\sum XY=a\sum X+b\sum X^2$

$\sum Y= 468$ $\sum X^2=42$

$\sum X= 0$ $\sum XY=166$ $N=8$

$\sum XY=a\sum X+b\sum X^2$

$\sum Y= Na+b\sum X$

$468=8(a)+b(0)$

$468=8a+0$

$468=8a$

$8a=468$

$a=468/8=58.5$

$166=a(0)+b(42)$

$166=0+42b$

$166=42b$

$42b=166$

$b=166/42=3.95$

Calculation of trend values

2012 a+Bx 58.5+3.95(-3.5) 58.5-13.825 =44.675	**2016** 58.5+3.95(0.5) 58.5+ 1.975= 60.475
2013 58.5+3.95(-2.5) 58.5-9.875= 48.625	2017 58.5+3.95(1.5) 58.5+5.925= 64.425
2014 58.5+3.95(-1.5) 58.5-5.925= 52.575	2018 58.5+3.95(2.5) 58.5+9.875= 68.375
2015 58.5+3.95(-0.5) 58.5-1.975= 56.525	2019 58.5+3.95(3.5) 58.5+13.825= 72.325

Example 2

- Calculate the trend values by taking 2015 as the mid year.

Year	Y
2012	40
2013	52
2014	60
2015	78
2016	35
2017	50
2018	63
2019	90

Year	Y	X	XY	X^2	Y_o
2012	40	-3	-120	9	44.675
2013	52	-2	-104	4	48.625
2014	60	-1	-60	1	52.575
2015	78	0	0	0	56.525
2016	35	1	35	1	60.475
2017	50	2	100	4	64.425
2018	63	3	189	9	68.375
2019	90	4	360	16	72.325
TOTAL	**468**	**4**	**400**	**44**	

Working note

- Y_o=a+bX y= 56.525+3.95(-3) =56.525-11.85 =44.675
- ∑Y= Na+b∑X

∑XY=a∑X+b∑X²

∑Y= 468 ∑X²=44

∑X= 4 ∑XY=400

468= 8(a)+4b

(400=4a+44b) -2

468 = 8a+4b

-800 = -8a-88b

-332=-84b

-84b=-332

b=-332/-84

b=3.95

468=8a+4b

468=8a +4(3.95)

468=8a+15.8

468-15.8=8a

8a=452.2

a=452.2/8

a=56.525

- 2013: 56.525+3.95(-2) = 56.525- 7.9 =48.625
- 2014: 56.525+3.95(-1) = 56.525-3.95 =52,575

2015:56.525+3.95(0) = 56.525-0 =56.525

- 2016:56.525+3.95(1) = 56.525+3.95 =60.475
- 2017:56.525+3.95(2) = 56.525+7.9 =64.425
- 2018:56.525+3.95(3) = 56.525+11.85 =68.375
- 2019:56.525+3.95(4) = 56.525+15.8 =72.325

Measures of dispersion - meaning

- An average itself does not tell us the full story
- We should know the manner in which other numbers scatter around.
- That is the reason we study dispersion

Different measures:

1. Range
2. Quartile deviation
3. Mean deviation
4. Standard deviation

students	Group A	Group B	Group C
1	50	45	30
2	50	50	45
3	50	55	75
Mean	50	50	50

Range

- The difference between the largest and the smallest values id called Range.
- Thus Range = L-S
- Coefficient of Range=

$$\frac{L-S}{L+S}$$

Examples

1. Find the range and the coefficient of range from the following. (individual series)

110, 117, 129, 197, 190, 100, 178, 255, 790

Largest item= 790

Smallest item= 100

R= L-S : 790-100=690

Coefficient of Range= $\frac{L-S}{L+S}$

$\frac{790-100}{790+100}$ =0.78

Calculate Range and its coefficient

Marks	No of students
10-20	08
2-30	10
30-40	12
40-50	08
50-60	04

Range= L-S

=60-10

=50

Coefficient of Range= $\frac{L-S}{L+S}$

$=\frac{60-10}{60+10}$ =0.714

Quartile deviation/Interquartile Range

$$QD = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Example 1

Find the value of QD and its coefficient from the following:

Roll No	1	2	3	4	5	6	7
Marks	20	28	40	12	30	15	50

Arranging the Marks (ascending order)
12
15
20
28
30
40
50

Q_1=Size of $\frac{N+1}{4}$ $\frac{7+1}{4}$ = 2nd Item

= 15

Q_3=Size of 3 $\left[\frac{N+1}{4}\right]$ 3 $\left\{\frac{7+1}{4}\right\}$ = 6th Item

= 40

$$QD = \frac{Q_3 - Q_1}{2}$$

$$QD = \frac{40 - 15}{2} = 12.5$$

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{40-15}{40+15}$$

$$= \frac{25}{55} = 0.455$$

Example 2.

Marks	**10**	**20**	**30**	**40**	**50**	**60**
No of students	04	07	15	08	07	02

Marks	**f**	**cf**
10	04	04
20	07	11
30	15	26
40	08	34
50	07	41
60	02	43

Q_1= Size of $\frac{N+1}{4}$ th item

- $\frac{43+1}{4}$ =11th item

Size of 11th item is 20

Thus Q_1=20

Q_3= Size of $3\frac{N+1}{4}$ th item

$=3\frac{43+1}{4}$ = 33rd item

Size of 33rd item is 40

Thus Q_3 = 40

QD= $\frac{Q_3-Q_1}{2}$

$=\frac{40-20}{2}$ =10

Coefficient of QD= $\frac{Q_3-Q_1}{Q_3+Q_1}$

$=\frac{40-20}{40+20}$

=0.333

- $Q_1 Q \left[\frac{Q-Q}{}\right]$

$$\left[\quad\right]$$

- $QD=\frac{Q_3-Q_1}{2}$
- $\frac{Q_3-Q_1}{2}$

- $QD=\frac{Q_3-Q_1}{2}$

Size of 33rd item is 40 Q_1= Size of N/4 th item =40+(0.7474)x5 =40+3.7373

Thus Q_3= 40

$QD=\frac{Q_3-Q_1}{2}=\frac{40-20}{2}$ =10 =200/4=50th item =35+

Coefficient of $QD=\frac{Q_3-Q_1}{Q_3+Q_1}=\frac{40-20}{40+20}$=0.333 Q_1 = L+ x i =4.005/79.4629 =0.0504

= 35+()

=35+0.726

=35.7256 Q_3 = Size of 3N/4 th item =43.7373 Coefficient of $QD= Q_3Q_3 - Q_1 - Q_1$ $\frac{N/4 - cf}{4}$

=3x200

4 Q_3= L+ xi

= 4.005

Example 3

- Find the co-efficient of quartile deviation for the following:

wages	No of employees	Cf
30-35	14	14
35-40	62	76
40-45	99	175
45-50	18	193
50-55	07	200

- $QD = \frac{Q_3 - Q_1}{2}$

Q_1= Size of N/4 th item

- Q_1 lies in the class 35-40

=200/4=50th item

$Q_1 = L + \frac{N/4 - cf}{f} \times i$

$= 35 + \frac{200/4 - 14}{62} \times 5$

$= 35 + (45/62)$

$= 35 + 0.726$

$= 35.7256$

Q_3 = Size of 3N/4 th item

$= \frac{3 \times 200}{4}$

=150th item

$Q_3 = L + \frac{3N/4 - cf}{f} \times i$

$Q_3 = 40 + (74/99) \times 5$

$= 40 + (0.7474) \times 5$

$= 40 + 3.7373$

$= 43.7373$

$QD = \frac{43.7373 - 35.7256}{2}$

$= 4.005$

Coefficient of QD= $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{43.7373 - 35.7256}{43.7373 + 35.7256}$

$= 4.005/79.4629$

$= 0.0504$

Module IV

- Ratio- Proportion- Direct proportion- inverse proportion
- Simple interest- True discount, bankers discount, bankers gain, equated due date, trade discount- cash discount
- Compound interest- formula for compound interest- interest payable half yearly or quarterly- interest for fraction of a period,
- Annuities- amount of an immediate annuity and annuity due, present value of annuity immediate and annuity due

Ratio and proportion

- The ratio is used for comparing two quantities that can be expressed in the same units.
- If a and b are the two quantities then the ratio is denoted by fraction as a/b and is expressed as a:b.
- Proportion:
- When two ratios are equal, the four quantities are said to be in proportion
- Eg: if a/b=c/d, a,b,c and d are in proportion.
- a:b=c:d or a:b::c:d
- a and d are called the extremes and b and c are called means.
- Product of the extremes is equal to product of the means.

Fourth proportion

- In a:b=c:d, d is the fourth proportion.
- If x is the fourth proportion can be written as,

a:b=c:x

a/b=c/x

The product of the extremes =product of the means,

ax=bc

X=bc/a

Third proportion

- If a/b=b/x or a:b=b:x, then x is the third proportion.
- a/b=b/x
- Hence the product of extremes =the product of the means,
- $ax=b^2$
- $X=b^2/a$

Mean proportion

- If a:x=x:b, the x is the mean proportion
- If a/x=x/b,
- Since the product of the extremes =product of the means,
- $ab=x^2$ or $x=\sqrt{ab}$
- $x^2=ab$ or x=

- Find the fourth proportion of 8:24=16:x

8:24=16:x

$$8(x) = 24 \times 16$$

$$x = \frac{24 \times 16}{8} = 384/8$$

$$x = 48$$

8:24=16:48 8x48=24x16 384=384

Find the third proportion to 6, 18, 18.

- If x is the third proportion,

  $6/18=18/x$ $\quad 6:18=18:x$

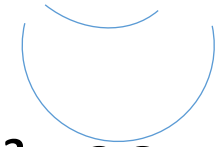
  $6x=18x18$

  $x=\frac{18x18}{6}=54$

  $6:18=18:54 \quad 6x54=18x18 \quad 324=324$

Find the mean proportion to 4, 9

$4:x=x:9$



$x^2 = 36$

$X=\sqrt{36}$

$=6$

$4:6=6:9$

$4x9=6x6$

$36=36$

4	**2**
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10

- 12x12=144
- 7/7= 1
- 3x2=6-4=2
- 22/7=3.14-0.14= 3
- 25=100/x 25xX=100 25X=100 X= 100/25=4
- 48/8= 6
- 7X7=49 7
- 2X2X2=8 √130===1301/26
- 100-89=11
- 2xX=3x6 ====2X=18 X=18/2 =9
- 5(6/3) 5(2) =10

1. A man gave 3/8 of his property to one son and 30% of the remainder to other. He then distributed the remaining property among 3 charities in the proportion of 2:5:7. if the differences of his son's share was Rs 4200, what was the value of his property and how much did each charity receive.

Ans:

Let the value of the property be X.

Share of one son=3/8 of X

Share of another son= (1-3/8)x30/100 $= \frac{1-3}{8} = \frac{8-3}{8} = \frac{5}{8}x$ LCM/Denominator xnumerator

$[\frac{5}{8}x]x\ 30/100 = \frac{30}{100}x\ [\frac{5}{8}\mathbf{x}] = \mathbf{\frac{3}{16}x}$

Difference of his son's share is Rs 4200

$\frac{3}{8}x - \frac{3}{16}x = 4200 = \frac{3}{8} - \frac{3}{16} = \frac{48-24}{128} = \frac{24}{128} = \frac{3}{16}x$

$\frac{3}{16}x = 4200 \quad x = \frac{4200}{3/16} = \frac{4200X16}{3}$ 22400

Property received by sons= $\frac{3}{8}x - \frac{3}{16}x = \frac{3}{8}$ = 8400 $\frac{3}{16}$x22400 =4200 8400+4200 = 12600

Property share left to charity= 22400-12600 =9800

The proportion is 2:5:7

First charity---- $\frac{2}{14}$x9800 =1400

Second charity $\frac{5}{14}$x9800 =3500

Third charity $\frac{7}{14}$ x 9800 = 4900 1400:3500:4900

Simple interest

- The reward given by the borrower to the lender is called interest.
- Interest is always paid as a percentage and is called rate of interest.
- There are two ways of calculating interest

1. Simple interest- interest is calculated for the sum of money (P)lent for a specific period (n) of time (t) at a given rate.(%)
2. Compound interest- interest is calculated at a given rate on both the principal amount and also the interest of the previous year.

Calculation of simple interest

- If the principal amount is P, and the period is N years at the rate r% per annum.
- The formula is,

  I= Pnr/100 ot I= Ptr/100

From the above equation,

$$p=\frac{100(I)}{nr} \qquad r=\frac{100(I)}{pn} \qquad n=\frac{100(1)}{Pr}$$

Eg:

- A merchant borrowed Rs 10000 on Jan 15, 2015 and repaid it with interest on 15, 2019 at 8% simple interest. What is the sum repaid by the merchant?
- Amount to be paid by the merchant= P+I
- I= $\frac{Ptr}{100}$ P= 10000 t= 4 years r= 8/100
- I=10000x4x$\frac{8}{100}$ =$\frac{320000}{100}$= 3200
- P+I = 10000+3200 =13200

Home work

- 1. a person has deposited Rs 10000 at 4% for a period of 3 years. Find the maturity value.
- 2. what is the simple interest on Rs 4000 for 8 years at 5%

1. What principal invested today will amount to Rs 2600 in 3 years at 10% simple interest?

amount after 3 years = P+I =2600 T= 3 Years r= 10%

2600= P + PTr/100

2600= P[1+Tr/100]

2600= P[1+3x10/100]

P[1+30/100] P[1+0.3]

P[1.3]

2600=P[1.3]

P[1.3]=2600

P=2600/1.3

P=2000

P+I =2600

2600= P+I

2600= 2000+I

2000+I=2600

I=2600-2000 = 600

True discount

- While quoting the price, the seller includes some interest for the period of credit and prepares the bill.
- The date on which the bill is due is known as due date or nominal due date.
- Normally a grace period of 3 days will be provided after the due date.
- This is called the legal due date.
- The interest charged on the present value for the due period is known as true discount.
- If P is the present value, r is the rate of interest, the true discount for a period of n years ,

$$TD = Pnr/100$$

The amount payable after n years is the face value of the bill or sum due is,

$$A = P + TD = P + Pnr/100 = p(1 + nr/100)$$

$$TD = A - P$$

Bankers discount

- When the bill is due after a certain period, it can be exchanged with a bank in advance.
- The bank charges some interest on the face value of the bill.
- The interest charged by the bank is called the bankers discount.
- Bankers discount BD=Anr/100
- A= face value of the bill, n= period, r=rate of interest
- Bankers Gain: the difference between banker's discount and true discount
- BG= BD-TD

- True discount= $\frac{ATR/100}{1+TR/100}$
- Bankers Discount= ATR/100
- Bankers Gain= BD-TD

Eg: 1

- A bill for Rs 8200 is due for 8 months. At 12% rate of interest, find the true discount, bankers discount and bankers gain.
- A=8200. n=8/12 r=12/100
- TD= $\frac{ATR/100}{1+TR/100}$ = $\frac{8200x\frac{8}{12}x\frac{12}{100}}{1+\frac{8}{12}x\frac{12}{100}}$ = 607.41
- BD= ATR/100 =$\frac{8200x8x0.12}{100}$ = 656
- BG= BD-TD =656-607.41 =48.59

2. home work

- A bill with face value of Rs.3000 is due after 3 months. It is discounted through a bank at 15%. Calculate the TD, BD, and BG.

Equated due date